

# Chapter 6: Data Mining for Temporal Data

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1 Introduction

2 Classification and clustering of time sequences

3 Time to event analysis

4 Analysis of Markov Chains

5 Association analysis

6 Sequence and episode mining

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# 1 Introduction

## – Basic ingredients

- A sequence of ordered time values, called observation times:  $t_1 \leq t_2 \leq \dots \leq t_T$
- Attribute values at these times:  $x_1 \leq x_2 \leq \dots \leq x_T$

## – Time sequence (time stamped data):

$$x = \langle (t_1, x_1), \dots, (t_T, x_T) \rangle$$

## – Two important goals in temporal data mining:

- Classification of time sequences into classes
- Finding clusters of time sequences

# 1 Introduction

## Time Sequences, Time Series, State Sequence

- A *time sequence* is defined as a sequence of time-stamped data for which the attribute values are the result of measurements of a quantitative real valued state variable  $y$ , i.e.,  $y \in \mathbb{R}$ . We denote the observations of a time sequence by  $y = (y(t_1), \dots, y(t_T))$
- A *time series* is a time sequence with equidistant predefined observation times denoted by  $y = (y_1, \dots, y_T)$
- A *state sequence* is a time sequence where the state variable  $S$  attains only a finite number of possible values given by a set  $\mathcal{S}$ . If the observation times are of minor importance, or even not known, we denote a chain simply as ordered sequence of observations of the state variable  $s = \langle s_1, \dots, s_T \rangle, s_i \in \mathcal{S}$

# 1 Introduction

## Event Set,

- Given a set  $\mathcal{E}$  of events an event set is subset  $E$  of  $\mathcal{E}$ .
- An event sequence is an ordered list of events  $s = \langle e_1, \dots, e_T \rangle$ . If the times of the events are known event sequences are denoted by  $s = \langle (e_1, t_1), \dots, (e_T, t_T) \rangle$ .

# 1 Introduction

## Problem formulation:

- A main issue is the representation of temporal data for the analysis
  - Non-adaptive representation: transform time sequence into a feature space, e.g. Fourier transformation
  - Adaptive representation: extract features of the time sequence, which can be used for analysis
  - Data clipping: transform time sequence into a bit string
  - Model based representation: time sequence as input for a model, e.g. a Markov chain
- We will focus on methods based on adaptive representation

# 1 Introduction

## Analysis Template

- *Relevant Business and Data*: Customer behavior represented as time sequence
- *Analytical Goals*:
  - Classification of a new time sequence into one of the possible classes
  - Segmentation of time sequences according to their structural similarity
- *Modeling Task*: Using visualization techniques for the time sequences of the process instances can support decision for a useful method:
  - Time warping for defining distances
  - Response features

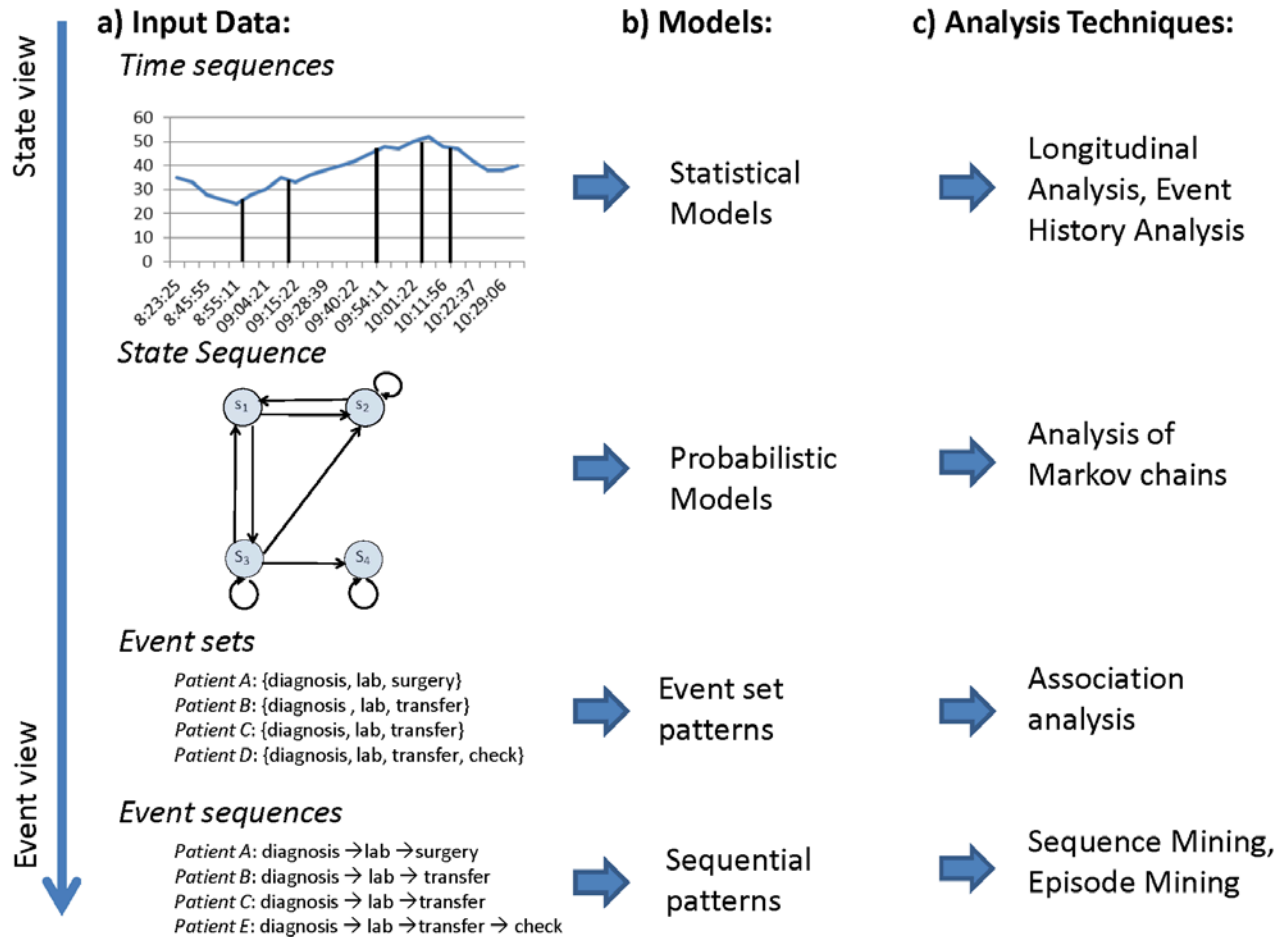
# 1 Introduction

## Analysis Template

- *Analysis task:*
  - Splitting data: If possible split the data randomly in one set for training and one set for validation
  - Model estimation: Estimate the warping path or the response features
  - Model Assessment: Assess quality of the model
  - Model Selection: Select a model
  - Use the results of model estimation for segmentation or classification
- *Evaluation and Reporting Task:* Evaluate the results of segmentation or classification either with test data or by using cross validation



# 1 Introduction



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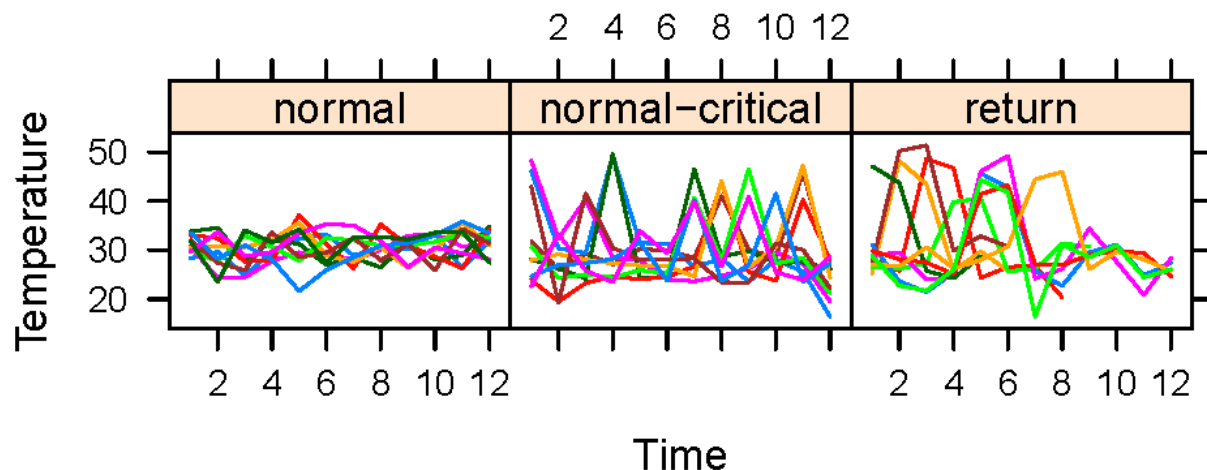
6 Sequence and episode mining

7 Summary and outlook

## 2 Classification and clustering of time sequences

### Dynamic Time Warping

- Example: Logistic Use Case
- The data show three different kinds of behavior:
  - A normal temperature regime
  - A critical temperature regime
  - A return temperature regime
- Goal is the identification of the regime



R package lattice

## 2 Classification and clustering of time sequences

### Classification based on time warping

- General Problem formulation:
- Given are data of customer behavior represented as time sequences for process instances
- These data are classified into different groups
- Task: Find a classification rule which allows the assignment of a time sequence to one of the classes

## 2 Classification and clustering of time sequences

### Classification based on time warping

- Basic idea behind time warping:
  - Classes are defined by time series which show a “similar pattern”
  - The term similarity is understood in the sense of speech waves: different persons spell words differently but we can classify the waves to words
- Problem which have to be taken into account:
  - Time sequences may have different length
  - Similarity may be blurred by some temporal transformations like stretching or squeezing some parts of the time sequence (see example)
  - We have to define the similarity by matching the observed values of two time sequences in such a way that the above defined effects are compensated

## 2 Classification and clustering of time sequences

### Classification based on time warping

- Dynamic time warping allows the calculation of similarity
- Basic is the definition of a warping path:

Given two sequences  $(x_1, \dots, x_N)$  and  $(y_1, \dots, y_M)$ :

Define a sequence  $(p_1, \dots, p_L)$  of matching indices pairs  $(i_l, j_l)$  such that

$$p_1 = (1,1) \quad p_L = (N, M)$$

$$(i_1 \leq i_2 \leq \dots \leq i_L) \text{ and } (j_1 \leq j_2 \leq \dots \leq j_L)$$

$$p_{l+1} - p_l \in \{(0,1), (1,0), (1,1)\}$$

- The last condition means that we increase the matching index at least by one step ahead

## 2 Classification and clustering of time sequences

### Classification based on time warping

- The costs of a warping path is defined by

$$DP = \sum_{l=1}^L d(i_l, j_l) = \sum_{l=1}^L |x_{i_l} - y_{j_l}|$$

- The dynamic time warping algorithm finds a warping path for two sequences with minimal costs
- The word “dynamic” indicates that the algorithm is based on dynamic programming

## 2 Classification and clustering of time sequences

### Classification based on time warping

- Application of the dynamic warping algorithm for all pairs of sequences defines a distance matrix for the observed time sequences
- We can apply now k-nearest neighbor classification for obtaining the classification rule



## 2 Classification and clustering of time sequences

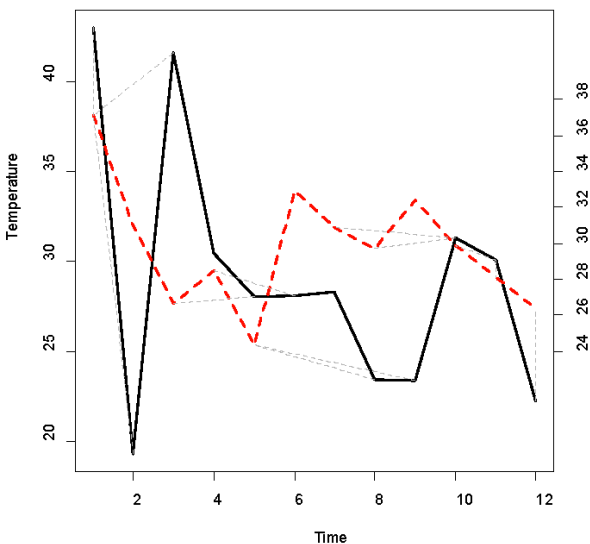
### Classification based on time warping

R package dtw

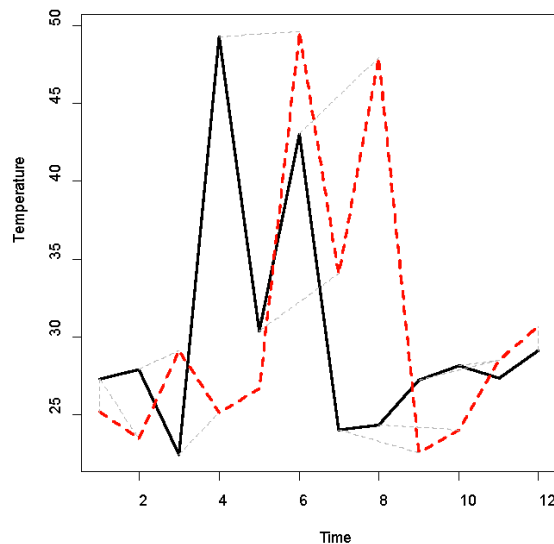
#### – Example: Logistics

- distance matrix between 100 time sequences defines the input for hierarchical clustering
- Ward method found 3 clusters: one with 50 normal correctly classified cases; one with 5 normal-critical cases and 5 return cases; one with 40 cases comprising 25 correctly classified return cases and 15 normal-critical cases

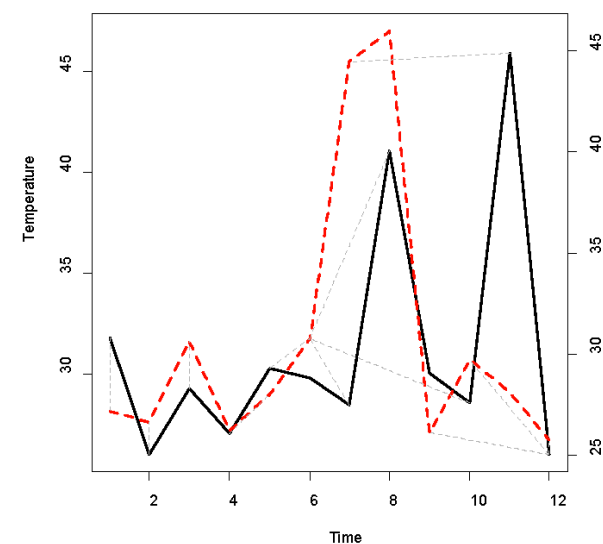
wrong normal



correct critical



wrong return



## 2 Classification and clustering of time sequences

### Classification Based on Response Features

- In that case we extract from the time sequence a number of time independent characteristic features
- Some examples of features:
  - Maximum and minimum of the time sequence
  - Temporal location of maximum and minimum
  - Breakpoints in the time sequence
  - Largest difference between two sequenced values
  - Length of the sequence
  - Area under the polygon defined by the sequence

## 2 Classification and clustering of time sequences

### Classification Based on Response Features

- More theoretically motivated features:
  - Transformation to frequencies and looking at the maximum frequency (Time sequence is sound or light)
  - Definition of a regression model for the time sequence
    - For equally spaced time measurements this can be done by time series analysis
    - For unequal spaced time measurements this is done by longitudinal data analysis
  - Definition of a representation language
- Based on these attributes one can apply methods of the classification of cross sectional data

## 2 Classification and clustering of time sequences

### Summary:

- Clustering of time sequences can be done using the same principles as in the case of classification
- The definition of time warping defines a distance for the sequences which can be used as input for cluster analysis (hierarchical or k-means)
- In the case of response features the distance between the time sequences is based on the definition of a distance for the response features

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## 3 Time to event analysis

### Problem formulation and terminology

- In Time-to-Event Analysis we are interested in modeling and predicting the time up to a certain event<sup>1,2</sup>
- Examples:
  - Prediction of the duration until a customer will quit her/his relationship with a company
  - Prediction of the duration of the lifetime of a certain device
- Other notions for such problems:
  - Event History Analysis
  - Survival Analysis

<sup>1</sup>G. Broström: Event History Analysis with R. CRC Press Taylor & Francis Group 2012

<sup>2</sup>R package `survival`

### 3 Time to event analysis

#### Problem formulation and terminology (ctd.)

- The time up to the event is called life time
- Main characteristic of the available data:
  - The data about the lifetime are **censored**, i.e., for some customers the event is observed, for others the event will occur in the future
  - This type of censoring is called **right censored**

## 3 Time to event analysis

### Terminology

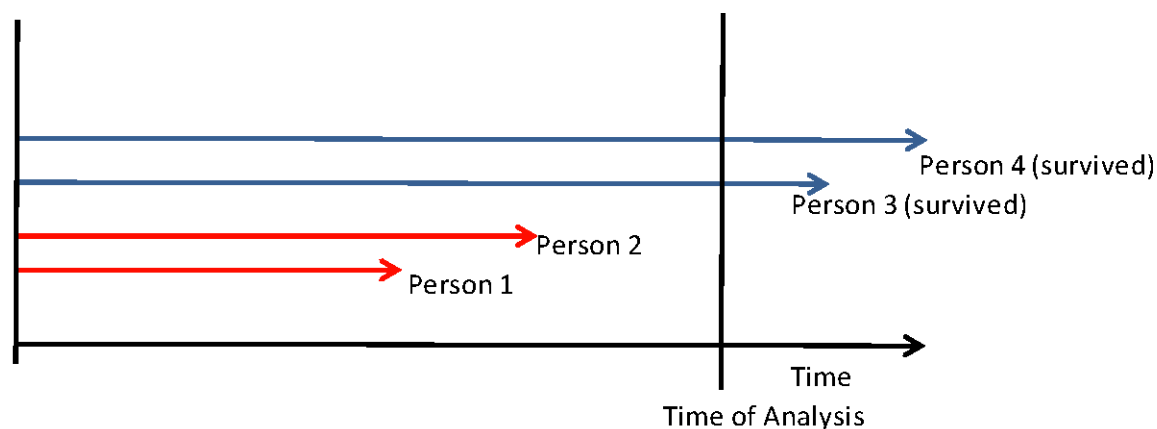
- The time up to the event is denoted by  $T$  and  $T$  is a random variable
- The probability that the event occurs before time  $t$  is denoted by  $F(t) = P(T \leq t)$
- The survival function is the probability that the event occurs after time  $t$ :  $S(t) = 1 - F(t)$
- The mean of the survival function is called the expected survival time
- The hazard function gives the likelihood that the event occurs at time  $t$ , given that the event has not occurred up to time  $t$ , formally:

$$h(t) = \frac{F'(t)}{1 - F(t)} = \frac{f(t)}{1 - F(t)}$$



### 3 Time to event analysis

- Graphical representation for two complete (red) and two censored (blue) lifetime observations



- Besides the censored lifetime usually other information about the customers is known, e.g. age, occupation, type of machine

### 3 Time to event analysis

#### Analysis template:

- *Relevant Business and Data*: Customer behavior represented by cross-sectional data and time sequences containing censored information about a terminal event
- *Analytical Goals*: Predict the duration up to the event for the censored time sequences from the uncensored data
- *Modeling Tasks*:
  - Definition of a survival table
  - Definition of a Cox regression model for the time to event
- *Analysis Tasks*:
  - Estimate the time up to the event using the Kaplan Meier estimate
  - Estimation of the coefficients in the Cox regression model
- *Evaluation and Reporting Task*: Evaluate the results using a method for the evaluation of regression

### 3 Time to event analysis

#### Modeling the survival function

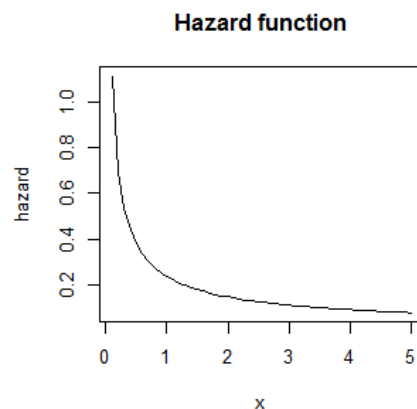
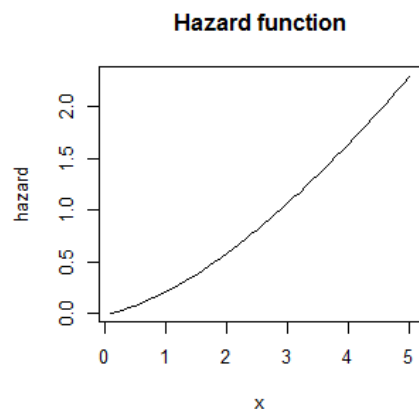
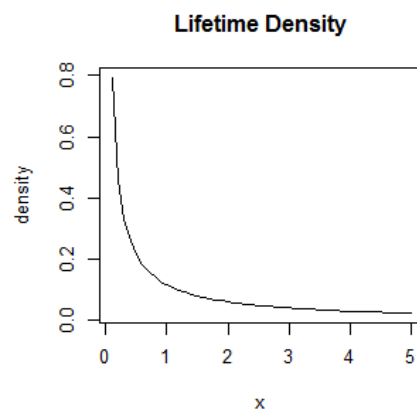
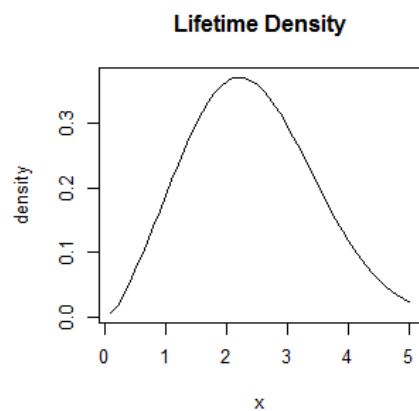
- A frequently used class of model in time-to-event analysis are Weibull distributions defined as:

$$F(t) = 1 - \exp[-(\alpha t)^\beta]$$
$$f(t) = \beta * (\alpha t)^{\beta-1} \alpha * \exp[-(\alpha t)^\beta]$$

- which allows adaptation to different scenarios like increasing hazard or decreasing hazard by choosing appropriate parameters

## 3 Time to event analysis

### – Examples of survival functions



R package graphics

### 3 Time to event analysis

#### Estimation of the survival function

- The basic information about the survival function is given by the Kaplan Meier estimate, which is summarized in the survival table with the following columns:
  - Time interval
  - Number of persons entering the interval (*n.risk*)
  - Number of events occurred in the interval (*n.event*)
  - Value of the survival function at the end of the time interval (*survival*)
  - The standard error of the estimate for the survival function
  - Confidence interval for the survival function

### 3 Time to event analysis

#### Example: survival table

- 305 patients with different types of melanoma observed from 2006 - 2010

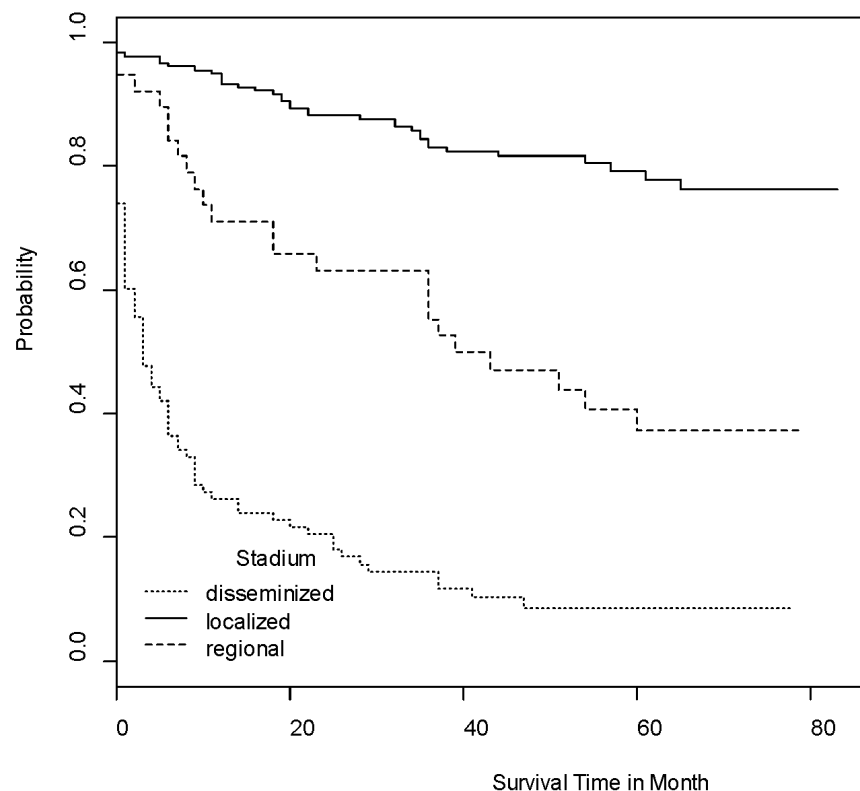
Year	n.risk	n.event	survival	std.err	lower 95% CI	upper 95% CI
0	305	69	0.774	0.0240	0.728	0.822
1	236	23	0.698	0.0263	0.649	0.752
2	213	19	0.636	0.0275	0.584	0.692
3	174	16	0.578	0.0286	0.524	0.637
4	136	6	0.552	0.0292	0.498	0.612
5	86	4	0.526	0.0305	0.470	0.590

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- Survival time can be plotted for groups of the population defined by some factors

### 3 Time to event analysis

- Example (ctd.): plot of survival function for the three groups



- survival functions for the three different values of stadium.
- disseminated cases have the worst prognosis for survival time and localized cases the best.

R package survival

## 3 Time to event analysis

### Cox Regression

- If there are additional explanatory variables for the occurrence of the event one can estimate the hazard rate with Cox regression, also known as proportional hazard model
- The model defines a time dependent baseline hazard for all observations which is modified according to the explanatory variables



## 3 Time to event analysis

### Cox Regression

- Estimation of the survival function, formally:

$$h(t) = h_0(t)\exp(\beta_1x_1 + \beta_2x_2 + \dots + \beta_kx_k)$$

- Interpretation of the parameters:
- For a quantitative explanatory variable  $x$  the relative risk changes by  $\exp(\beta)$  if  $x$  is increased by one unit
- For a dummy variable representing a factor level the relative risk changes by compared to a reference level
- Example:
- For the 305 patients the influence of the explanatory variables age at diagnosis and stadium of the tumor is of interest
- The results are shown on the next slide

### 3 Time to event analysis

```
coxph(formula = Surv(Time, Event) ~ Age_Diagnosis + Stadium,  
      data = vie1)
```

```
n= 305, number of events= 137
```

	coef	exp(coef)	se(coef)	z	Pr(> z )	
Age_Diagnosis	0.02991	1.03036	0.00653	4.580	4.64e-06	***
Stadiumlocalized	-2.64494	0.07101	0.21324	-12.404	< 2e-16	***
Stadiumregional	-1.41158	0.24376	0.24521	-5.756	8.59e-09	***
Stadiumunknown	NA	NA	0.00000	NA	NA	

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

	exp(coef)	exp(-coef)	lower .95	upper .95
Age_Diagnosis	1.03036	0.9705	1.01726	1.0436
Stadiumlocalized	0.07101	14.0826	0.04675	0.1079
Stadiumunknown	NA	NA	NA	NA

```
Concordance= 0.835 (se = 0.027 )  
Rsquare= 0.456 (max possible= 0.992 )  
Likelihood ratio test= 185.5 on 3 df, p=0  
Wald test = 166.9 on 3 df, p=0  
Score (logrank) test = 236.6 on 3 df, p=0
```

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## 4 Analysis of Markov Chains

Two representations:

– Probabilistic representation:

- Stochastic matrix  $P = (p_{ij})$  defined by the transition probabilities from state  $s_i$  to state  $s_j$  in one time step.
- All entries are positive and the rows sum up to one.
- Transition matrix after  $n$  steps is denoted by  $P(n)$ . By using the Chapman-Kolmogorov equations,  $P(n)$  can be calculated by matrix multiplication, i.e.,  $P(n) = P^n$ .
- If we denote the initial probabilities for the possible states at  $t = 0$  by  $\mu_0^{(i)} = P(S_0 = si)$  and by  $\mu_n^{(i)}$  the probabilities of the states at time  $T = n$  we can calculate the probabilities of the different states after  $n$  time steps by  
$$\mu_n = \mu_0 * P^n, \mu_0 = (\mu_0^{(1)}, \dots, \mu_0^{(K)})$$

– Graphical representation obtained by interpreting the matrix of transition probabilities as weighted adjacency matrix of a directed graph with nodes defined by the states of the process.

## 4 Analysis of Markov Chains

Analysis template:

- *Relevant Business and Data*: Process instances represented as states or event sequences
- *Analytical Goals*:
  - Estimation of state transitions from exiting instances
  - Structural behavior of state transitions in the long run
  - Segmentation of sequences into groups
  - Segmentation of the states
- *Modeling Tasks*: Definition of a stationary Markov chain for state transitions
- *Analysis Tasks*:
  - Estimation of transition probabilities
  - Estimate of a stable distribution
  - Cluster analysis for instances of state or event sequences
  - Cluster analysis of the states or events
- *Evaluation and Reporting Task*: cf. Chapter 5

## 4 Analysis of Markov Chains

Estimation problems: based on structural analysis of Markov chains

- Goal: finding transition probabilities in the long run
- Important: classification of the states of a Markov chain with respect to the transition behavior
- Basis: typology of states based on *graph representation*
  - state  $s_i$  is *reachable* from state  $s_j$  if there is a path from  $s_i$  to  $s_j$ ;  $(s_i, s_j)$  defines a path of length 1 (then:  $s_i, s_j$  directly linked)
  - $s_i, s_j$  are *connected* if  $s_i$  is reachable from  $s_j$  and vice versa,
  - connected states define a *closed path*
  - *connectedness* defines a partition of all states into classes of connected states
  - A Markov chain is called *irreducible* if each state can be reached from any other state in finite time, i.e., all states belong to one class.
  - Closed set of states as states which cannot be left as soon as we have reached the states.
  - An *absorbing* state is a closed state not connected to any other state. For an absorbing state  $s_i$ , we have  $p_{ij} = 1$ .

## 4 Analysis of Markov Chains

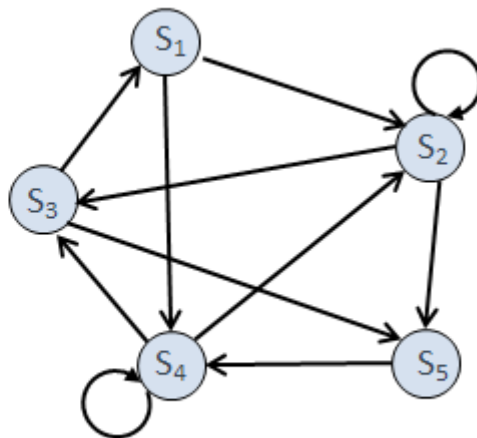
### Estimation problems:

- A state is called *transient* if there is a positive probability of not returning into the state.
- A state is called *recurrent* if the probability of returning into the state is 1.
- In the case of irreducible Markov chains, all states are either recurrent or transient.
- For recurrent states we can define the period as the largest common divisor of all times  $t$  for which  $p_{ii}(n) > 0$ .
- If the period of a state is 1, the state is called *aperiodic*.
- A Markov chain where all states are aperiodic is called *ergodic*.

## 4 Analysis of Markov Chains

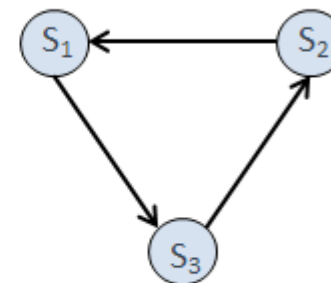
- Left side (a): ergodic
- Right side (b): length of period = 3

	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$
$S_1$	0	0.6	0	0.4	0
$S_2$	0	0.3	0.4	0	0.3
$S_3$	0.6	0	0	0	0.4
$S_4$	0	0.4	0.5	0.1	0
$S_5$	0	0	0	1.0	0



a) Irreducible Markov Chain

	$S_1$	$S_2$	$S_3$
$S_1$	0	0	1
$S_2$	1	0	0
$S_3$	0	1	0



b) Periodic Markov Chain

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## 4 Analysis of Markov Chains

Estimation of transition probabilities:

- *Given:*  $N$  state sequences  $s_1, s_2, \dots, s_N$  of possibly different length, generated by a homogeneous Markov chain with  $K$  states  
 $s_1, s_2, \dots, s_K$
- *Goal:* estimate the transition probabilities  $p_{ij}$
- $p_{ij} \neq 0$  only for those transitions for which an edge between the vertices exists in the graph representation.
- If all transitions are generated independently, the distribution of the number of transitions from a state  $s_i$  to its directly linked states is a *multinomial* distribution.
- Given a number of state sequences, we denote by  $n_{ij}$  the observed number of one step transitions from state  $s_i$  to state  $s_j$  and by  $n_i$  the observed number of occurrences of state  $s_i$ .

## 4 Analysis of Markov Chains

Estimation of transition probabilities – methods:

- *Maximum likelihood* estimate:  $\hat{p}_{ij} = \frac{n_{ij}}{n_i}$
- $\rightarrow$  if transition from  $s_i$  to  $s_j$  is not observed,  $\hat{p}_{ij} = 0$  though it might be possible from the structure of the Markov chain
- *Bayesian approach*: prior distribution is assumed; estimates are calculated as means of posterior distribution
  - Prior Dirichlet distribution:  $P(p_{i1}, p_{i2}, \dots, p_{iK}) = C \prod p_{ij}^{\alpha_{ij}-1}$ ,  $\alpha_i > 0$  called concentrations
  - Estimates:  $\tilde{p}_{ij} = \frac{n_{ij} + \alpha_{ij}}{n_i + \alpha_0}$
  - Example: prediction of page requests on the Internet;  $\alpha_j$  relations between outgoing links of the pages;  $\alpha_0$  number of observations necessary for substantial change of prior beliefs

## 4 Analysis of Markov Chains

### Cluster analysis for Markov chains:

- Goal: finding groups of Markov chains with similar structure based on cluster algorithm<sup>3</sup>
- Idea:
  - Interpret event sequence  $s$  as Markov chain  $M$  (events as states  $s_i$ )
  - Assign probability that event sequence is generated by a Markov chain, following Maximum likelihood estimate (previous slide):
  - $P(s | M) = \pi(s_1) * \prod P_M(s_i | s_k)$  with
  - $P_M$  transition probabilities of Markov chain  $M$  and  $\pi_M$  initial probability
  - Clustering of Markov chains based on such transition probabilities resembling k-means clustering

<sup>3</sup>Rebuge A, Ferreira DR (2012) Business process analysis in health care environments: A methodology based on process mining. Information Systems 37(2):99–116

## 4 Analysis of Markov Chains

### Cluster analysis for Markov chains:

- Example:
- $\mathcal{S} = \{\text{CN}, \text{CP}, \text{HN}, \text{HP}, \text{EX}, \text{start}, \text{end}\}$
- Event sequence  $\langle \text{start}, \text{CN}, \text{CN}, \text{CP}, \text{HN}, \text{CN}, \text{CP}, \text{HP}, \text{EX}, \text{end} \rangle$
- This generates a Markov chain with probabilities
  - $P(\text{CN} \mid \text{start}) = 1$
  - $P(\text{CN} \mid \text{CN}) = 1/3, P(\text{CP} \mid \text{CN}) = 2/3$
  - $P(\text{HN} \mid \text{CP}) = P(\text{HP} \mid \text{CP}) = 1/2$
  - $P(\text{CN} \mid \text{HN}) = P(\text{EX} \mid \text{HP}) = P(\text{end} \mid \text{EX}) = 1$

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## 5 Association analysis

– *Input*<sup>4</sup>:

- Set of items  $I := \{i_1, \dots, i_n\}$
- $T$  defines a set of transactions; each transaction  $t \in T$  is defined as a vector  $t := \langle t[1], \dots, t[n] \rangle$  with  $t[j] = 1$  if item  $i_j$  is associated with  $t$  and  $t[j] = 0$  otherwise
- $X \subseteq I$  denotes the item set of interest, i.e., we are looking for rules  $X \Rightarrow I_j$  with  $I_j \in I$  and  $I_j \notin X$
- $t \in T$  satisfies  $X$  if  $\forall x \in X: t[x] = 1$
- *Explanation*: the goal is to find association rules  $A \Rightarrow B$  where the occurrence of  $A$  implies the occurrence of  $B$ ;  $A$  is called the antecedent,  $B$  the consequent of the rule

<sup>4</sup>Agrawal R, Imielinski T, Swami A (1993) Mining association rules between sets of items in large databases. ACM SIGMOD Record 22(2):207–216

## 5 Association analysis

### Example (Hospital):

- Transaction 1:
  - Event 1 = Prescribe Aspirin
  - Event 2 = Prescribe Marcumar
- Transaction 2:
  - Event 1 = Prescribe Aspirin
  - Event 2 = Prescribe Marcumar
- Transaction 3:
  - Event 1 = Prescribe Aspirin
  - Event 2 = Prescribe Paracetamol
- Transaction 4:
  - Event 1 = Prescribe Aspirin
  - Event 2 = Prescribe Marcumar

## 5 Association analysis

Example (Hospital), ctd.

- Assume item set  $I = \{\text{Aspirin, Marcumar, Paracetamol}\}$

Transaction	Aspirin	Marcumar	Paracetamol
$t_1 = \langle 1, 1, 0 \rangle$	1	1	0
$t_2 = \langle 1, 1, 0 \rangle$	1	1	0
$t_3 = \langle 1, 0, 1 \rangle$	1	0	1
$t_4 = \langle 1, 1, 0 \rangle$	1	1	0

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- All transactions support item set  $\{\text{Aspirin}\}$



## 5 Association analysis

### Confidence of Association Rule

- Let  $T$  be a set of transactions,  $A \subseteq I$  be an item set of interest and  $B \in I$  be an item set. Then the confidence  $c$  of a rule:  $A \Rightarrow B$  is defined as follows:  $c(R, T) := \frac{|\{t \text{ with } t \text{ satisfies } A \cup B\}|}{|A|}$
- Confidence enables to measure the rule strength
- *Example:*  $R: \text{Aspirin} \Rightarrow \text{Marcumar}; c(R, T) = 0.75$

### Support of Association Reuls

- Let  $T$  be a transaction set and  $R: A \Rightarrow B$  a rule. Then the support  $s(R, T)$  is defined as  $s(R, T) := \frac{|\{t \text{ with } t \text{ satisfies } \{A, B\}\}|}{|T|}$
- *Example:*  $R: \text{Aspirin} \Rightarrow \text{Marcumar}; s(R, T) = 0.75$

It holds:  $c(R, T) = \frac{s(R, T)}{s(A \Rightarrow, T)}$

## 5 Association analysis

### Main Steps in Association Analysis:

- Find large item sets:
  - Define minimal support
  - Find all time sets, for which their support exceeds the threshold (-→ large item sets)
- Discover rules within large item sets:
  - Define minimal confidence
  - Determine all possible rules in the large item sets that exceed confidence and adhere additional syntactical constraints
- *Example*: minimal support = 0.2; minimal confidence: 0.5; syntactical constraints: antecedent must contain Aspirin consequent not empty
  - Large item sets: {A}, {M}, {P}, {A, M}, {A, P}
  - Rules in large item sets with  $c > 0.2$ ; R1:  $A \Rightarrow M$  and R2:  $A \Rightarrow P$
  - Result: R1 with  $c(R1, T) = 0.75$

## 5 Association analysis

Easy Miner: <http://www.easyminer.eu/>

[< Back](#)

### Upload CSV File

#### Import config

Uploaded CSV file: AssociationMining\_Ex1.csv

Table name:

Encoding:

Separated by:

Text delimiter:

Escape character:

#### Import statistics

Rows count: 4

Columns count: 3


[Import data into database...](#)

#### Data preview


aspirin <i>int</i>	marcumar <i>int</i>	paracetamol <i>int</i>
1	1	0
1	1	0
1	0	1
1	1	0

## Association rule pattern

<u>Antecedent</u>	<u>Interest measures</u>	<u>Consequent</u>
aspirin (*)	Confidence: 0.7 Support: 0.25 <a href="#">+ Add IM</a>	marcumar (*)

 Do you want to [mine rules?](#)

## Discovered rules

 Mining has finished!

< 1 >

1. aspirin (1) => marcumar (1)  
[Confidence: 0.750000, Support: 0.750000]

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## 6 Sequence and episode mining

### Medical example

<b>Aspirin</b>	<b>BetaBlock</b>	<b>Ibu</b>	<b>Antibiotics</b>	<b>Time stamp</b>	<b>Patient</b>
1	0	0	0	10.10.2013	P1
1	0	1	0	12.10.2013	P2
0	0	0	1	13.10.2013	P2
0	1	0	0	14.10.2013	P1
1	0	0	0	15.10.2013	P3
0	0	1	0	16.10.2013	P3
1	0	1	0	17.10.2013	P4
0	1	0	1	18.10.2013	P4

<b>Patient ID</b>	<b>Item set</b>	<b>Sequence</b>
P1	{Aspirin, BetaBlock}	<Aspirin, BetaBlock>
P2	{Aspirin, Ibu, Antibiotics}	<Aspirin, Ibu, Antibiotics>
P3	{Aspirin, Ibu}	<Aspirin, Ibu>
P4	{Aspirin, Ibu, BetaBlock, Antibiotics}	<Aspirin, Ibu, BetaBlock, Antibiotics>

## 6 Sequence and episode mining

### Sequence mining<sup>5</sup>

- So far: mining of rules, e.g., Aspirin  $\Rightarrow$  {Ibu, Antibiotics} with support 0.5 and confidence 0.5
- Not known: order of Ibu and Antibiotics
- Goal of sequence mining:
- If Aspirin, then Ibu followed by Antibiotics

<sup>5</sup>Agrawal R, Srikant R (1995) Mining sequential patterns. In: Yu PS, Chen ALP (eds) ICDE'95: International Conference on Data Engineering, IEEE, pp. 3–14

## 6 Sequence and episode mining

Sequence mining – input:

- $\mathcal{I} := \{i_1, \dots, i_n\}$  defines the set of items
- $T$  defines a set of transactions; a time stamp is assigned to each transactions
- $S := \langle s_1, \dots, s_k \rangle$  denotes a sequence of item sets
- A sequence  $S$  is contained in another sequence  $S'$ , i.e.,  $S < S'$  if  $\forall s \in S: \exists s' \in S' \text{ with } s \subseteq s'$
- $T$  is called *customer sequence*; it constitutes an ordered sequence of transactions referring to item sets, i.e., it is a item set itself
- *Example sequences*:  $S1 = \langle \{\text{Aspirin}\}, \{\text{BetaBlock}\} \rangle$ ,  
 $S2 = \langle \{\text{Aspirin}, \text{Ibu}\}, \{\text{BetaBlock}, \text{Antibiotics}\} \rangle$  with  $S1 < S2$

## 6 Sequence and episode mining

Sequence mining – analytical goal:

- *Find the maximum sequences in the customer sequence with a user-defined minimum support.*
- Sub task 1: finding sequences with minimum support → large sequences
- Sub task 2: out of them finding the maximum ones
- Let  $C$  be a set of customers and  $S$  be a sequence. Then  $c$  in  $C$  supports  $S$  if  $S$  is contained in  $S_c$  with  $S_c$  being the sequence of customer  $c$ :  
$$s(S, C) = \frac{|\{c \text{ with } S \prec S_c\}|}{|C|}$$
- cf. finding large item sets
- *length(s)*: number of items within sequence  $s$
- $S$  maximal if  $\nexists S' \text{ with } S \prec S' \text{ and } \text{length}(S) \leq \text{length}(S')$



## 6 Sequence and episode mining

### Sequence mining – Example (ctd.)

- Assume minimum support of 0.4
- We find the following large sequences:
  - S1 = <Aspirin> with support 1 and length 1
  - S2 = <Ibu> with support 0.5 and length 1
  - S3 = <BetaBlock> with support 0.5 and length 1
  - S4 = <AntiBiotics> with support 0.5 and length 1
  - S5 = <Aspirin, BetaBlock> with support 0.5 and length 2
  - S6 = <Aspirin, AntiBiotics> with support 0.5 and length 2
  - S7 = <Aspirin, Ibu, AntiBiotics> with support 0.5 and length 3

Implementations of algorithms available at: <http://www.philippe-fournier-viger.com/spmf/>

## 6 Sequence and episode mining

### Sequence versus episode mining<sup>6</sup>

- Input data:
  - Sequence mining: transactional set with time stamps
  - Episode mining: stream of time-stamped events
- Pattern structure:
  - Sequence mining: maximum sequences of item sets
  - Episode mining: partly ordered collection of events occurring together
- Distinction between serial and parallel episodes
- Example:  $s = \langle (A, 2), (M, 3), (A, 4), (B, 5), (A, 8), (M, 9), (B, 10), (I, 12), (A, 13), (A, 15), (M, 16), (B, 18), (A, 19) \rangle$
- Serial episode: A is always followed by M
- Parallel episode: A and B frequently occur together

<sup>6</sup>Mannila H, Toivonen H, Verkamo IA (1997) Discovery of frequent episodes in event sequences. Data Mining and Knowledge Discovery 1(3):259–289

## 6 Sequence and episode mining

### Episode mining – definitions:

- Goal: find the neighborhood where potential patterns occur in
- Occurrence must be confined to a segment of the event stream
- → use of a certain size to subdivide the event stream
- Let  $E$  be an event set. Assume an event sequence  $s := \langle (e_1, t_1), \dots, (e_n, t_n) \rangle$  where  $e_i$  are the events and  $t_i$  the associated time stamps with  $t_i \leq t_{i+1}$ . Then an episode  $\varepsilon := (V, \leq, g)$  with  $v$  being a set of nodes,  $\leq$  being a partial order and  $g$  being a mapping function  $g: V \rightarrow E$ .
- Episodes are represented by graphs where edges represent patterns. Edge(A,B) represented a serial pattern between events A and B; if A and B parallel, no edges between A and B exists.

## 6 Sequence and episode mining

### Episode mining – definitions (ctd.)

- Window  $w := (s, t_s^w, t_e^w)$  with  $t_s^w < t_n, t_s^w > t_1$
- Window width:  $width(w) := t_e^w - t_s^w$
- Set  $W(s, ws) := \{w \text{ over } s \text{ with } width(w) = ws\}$
- How often does a given episode  $\varepsilon$  occur for windows of size  $ws$  on sequence  $s$ ? Frequency for this calculated as follows:

$$f(\varepsilon, s, ws) := \frac{|\{w \in W(s, ws) \text{ with } \varepsilon \text{ occurs in } w\}|}{|W(s, ws)|}$$

- Define minimum frequency threshold
- Frequent episodes exceed frequency

## 6 Sequence and episode mining

### Episode mining – example (ctd.)

- Window  $w=(s, 3, 5)$  containing event occurrences A, M, A.  
Overall, s contains 9 windows of size 3, i.e.:
- $W(s,3) = \{ (A, M, A), (M, A, B), (A, B, A), (B, A, M), (A, M, B), (M, B, I), (B, I, A), (I, A, A), (A, A, M) \}$
- Assume episode  $\varepsilon=(\{v1, v2\}, \leq, g)$  with  $g(v1)=A$  and  $g(v2)=M$ .  $\varepsilon$  is a sequential episode as we are looking for patterns where event A precedes event M.
- $f(\varepsilon, s, ws) = 4/9 \approx 0.44$

*Analytical goal:* find all frequent episodes!

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## 7 Summary and outlook

- A lot of data is temporal
- Newly arising scenarios: machining data with time stamps
- Finding series or patterns in such data can be very interesting for analysis and predictions
- Finding patterns, sequence, and episodes already paves the way to process mining (→ Chapter 7)

## 7 Summary & outlook

